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GRAPHS WHOSE A_α-SPECTRAL RADIUS DOES NOT EXCEED 2

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This paper is dedicated to the memory of our excellent colleague Slobodan K. Simić who recently passed away.

Abstract

Let $A(G)$ and $D(G)$ be the adjacency matrix and the degree matrix of a graph G, respectively. For any real $\alpha \in [0,1]$, we consider $A_\alpha(G) = \alpha D(G) + (1-\alpha)A(G)$ as a graph matrix, whose largest eigenvalue is called the A_α-spectral radius of G. We first show that the smallest limit point for the A_α-spectral radius of graphs is 2, and then we characterize the connected graphs whose A_α-spectral radius is at most 2. Finally, we show that all such graphs, with four exceptions, are determined by their A_α-spectra.

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